Calculus 1
Workbook Solutions

Definition of the derivative

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MATH

DIFFERENCE QUOTIENT

■ 1. Evaluate the difference quotient for $f(x) = -9x + 10$ at $x = -6$.

Solution:

The difference quotient of the function $f(x)$ at $x = a$ is

$$\frac{f(a+h) - f(a)}{h}$$

$$\frac{f(-6+h) - f(-6)}{h}$$

$$\frac{(-9(-6+h) + 10) - (-9(-6) + 10)}{h}$$

$$\frac{(54 - 9h + 10) - (54 + 10)}{h}$$

$$\frac{54 - 9h + 10 - 54 - 10}{h}$$

$$\frac{-9h}{h}$$

$$-9$$

■ 2. Evaluate the difference quotient for $g(x) = -10x^2 + 7$ at $x = 3$.

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Solution:

The difference quotient of a function $g(x)$ at $x = a$ is

$$\frac{g(a+h)-g(a)}{h}$$

$$\frac{g(3+h)-g(3)}{h}$$

$$\frac{(-10(3+h)^2+7)-(-10(3)^2+7)}{h}$$

$$\frac{(-10(9+6h+h^2)+7)-(-10(9)+7)}{h}$$

$$\frac{-90-60h-10h^2+7+90-7}{h}$$

$$\frac{-60h-10h^2}{h}$$

$$-60-10h$$

■ 3. Evaluate the difference quotient for $h(x) = 2x^3 + 5x$ at $x = -2$.

Solution:

The difference quotient of a function $h(x)$ at $x = a$ is

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$$\frac{h(a+h)-h(a)}{h}$$

$$\frac{h(-2+h)-h(-2)}{h}$$

$$\frac{(2(-2+h)^3+5(-2+h))-(2(-2)^3+5(-2))}{h}$$

$$\frac{-16+16h-4h^2+8h-8h^2+2h^3-10+5h+16+10}{h}$$

$$\frac{2h^3-12h^2+29h}{h}$$

$$2h^2-12h+29$$

4. Evaluate the difference quotient for $f(x)=5x^2-6x$ at $x=4$.

Solution:

The difference quotient of a function $f(x)$ at $x=a$ is

$$\frac{f(a+h)-f(a)}{h}$$

$$\frac{f(4+h)-f(4)}{h}$$

$$\frac{(5(4+h)^2-6(4+h))-(5(4)^2-6(4))}{h}$$

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$$\frac{(5(16+8h+h^2)-24-6h)-(80-24)}{h}$$

$$\frac{80+40h+5h^2-24-6h-80+24}{h}$$

$$\frac{34h+5h^2}{h}$$

$$34+5h$$

■ 5. Evaluate the difference quotient for $g(x)=9x^2+3x$ at $x=-1$.

Solution:

The difference quotient of a function $g(x)$ at $x=a$ is

$$\frac{g(a+h)-g(a)}{h}$$

$$\frac{g(-1+h)-g(-1)}{h}$$

$$\frac{(9(-1+h)^2+3(-1+h)-(9(-1)^2+3(-1))}{h}$$

$$\frac{(9(1-2h+h^2)-3+3h)-(9-3)}{h}$$

$$\frac{9-18h+9h^2-3+3h-9+3}{h}$$

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$$\frac{-15h + 9h^2}{h}$$

$$-15 + 9h$$

■ 6. Evaluate the difference quotient for $h(x) = -2x^2 + 8x - 4$ at $x = 1$.

Solution:

The difference quotient of a function $h(x)$ at $x = a$ is

$$\frac{h(a+h) - h(a)}{h}$$

$$\frac{h(1+h) - h(1)}{h}$$

$$\frac{(-2(1+h)^2 + 8(1+h) - 4) - (-2(1)^2 + 8(1) - 4)}{h}$$

$$\frac{(-2(1+2h+h^2) + 8 + 8h - 4) - (-2 + 8 - 4)}{h}$$

$$\frac{-2 - 4h - 2h^2 + 8 + 8h - 4 + 2 - 8 + 4}{h}$$

$$\frac{4h - 2h^2}{h}$$

$$4 - 2h$$

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DEFINITION OF THE DERIVATIVE

■ 1. Use the definition of the derivative to find the derivative of $f(x) = 2x^2 + 2x - 12$ at $(4,28)$.

Solution:

The definition of the derivative is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

So at $(4,28)$, the derivative will be

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

$$f'(4) = \lim_{h \to 0} \frac{(2(4+h)^2 + 2(4+h) - 12) - (2(4)^2 + 2(4) - 12)}{h}$$

$$f'(4) = \lim_{h \to 0} \frac{(2(16 + 8h + h^2) + 8 + 2h - 12) - (32 + 8 - 12)}{h}$$

$$f'(4) = \lim_{h \to 0} \frac{32 + 16h + 2h^2 + 8 + 2h - 12 - 32 - 8 + 12}{h}$$

$$f'(4) = \lim_{h \to 0} \frac{18h + 2h^2}{h}$$

$$f'(4) = \lim_{h \to 0} 18 + 2h$$

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Evaluate the limit to find the derivative at $(4,28)$.

$$f'(4) = 18 + 2(0)$$

$$f'(4) = 18$$

■ 2. Use the definition of the derivative to find the derivative of $g(x) = 3x^3 - 4x + 7$ at $(-2, -9)$.

Solution:

The definition of the derivative is

$$g'(x) = \lim_{x \to a} \frac{g(x) - g(a)}{x - a}$$

So at $(-2, -9)$, the derivative will be

$$g'(-2) = \lim_{x \to -2} \frac{g(x) - g(-2)}{x - (-2)}$$

$$g'(-2) = \lim_{x \to -2} \frac{(3x^3 - 4x + 7) - (3(-2)^3 - 4(-2) + 7)}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{(3x^3 - 4x + 7) - (-24 + 8 + 7)}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{3x^3 - 4x + 16}{x + 2}$$

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$$g'(-2) = \lim_{x \to -2} \frac{(x+2)(3x^2 - 6x + 8)}{x+2}$$

$$g'(-2) = \lim_{x \to -2} 3x^2 - 6x + 8$$

Evaluate the limit to find the derivative at $(-2, -9)$.

$$g'(-2) = 3(-2)^2 - 6(-2) + 8$$

$$g'(-2) = 32$$

■ 3. Use the definition of the derivative to find the derivative of $h(x) = 9x^2 - 7x - 4$ at $(2,18)$.

Solution:

The definition of the derivative is

$$h'(x) = \lim_{h \to 0} \frac{h(x+h) - h(x)}{h}$$

So at $(2,18)$, the derivative will be

$$h'(2) = \lim_{h \to 0} \frac{h(2+h) - h(2)}{h}$$

$$h'(2) = \lim_{h \to 0} \frac{(9(2+h)^2 - 7(2+h) - 4) - (9(2)^2 - 7(2) - 4)}{h}$$

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$$h'(2) = \lim_{h \to 0} \frac{(9(4 + 4h + h^2) - 14 - 7h - 4) - (36 - 14 - 4)}{h}$$

$$h'(2) = \lim_{h \to 0} \frac{36 + 36h + 9h^2 - 14 - 7h - 4 - 36 + 14 + 4}{h}$$

$$h'(2) = \lim_{h \to 0} \frac{29h + 9h^2}{h}$$

$$h'(2) = \lim_{h \to 0} 29 + 9h$$

Evaluate the limit to find the derivative at $(2,18)$.

$$h'(2) = 29 + 9(0)$$

$$h'(2) = 29$$

4. Use the definition of the derivative to find the derivative of $h(x) = 8x^2 - 19x + 15$ at $(2,9)$.

Solution:

The definition of the derivative is

$$h'(x) = \lim_{x \to a} \frac{h(x) - h(a)}{x - a}$$

So at $(2,9)$, the derivative will be

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$$h'(2) = \lim_{x \to 2} \frac{h(x) - h(2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{(8x^2 - 19x + 7) - (8(2)^2 - 19(2) + 7)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{8x^2 - 19x + 7 - 32 + 38 - 7}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{8x^2 - 19x + 6}{x + 2}$$

$$h'(2) = \lim_{x \to 2} \frac{(8x - 3)(x - 2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} 8x - 3$$

Evaluate the limit to find the derivative at $(2,9)$.

$$h'(2) = 8(2) - 3$$

$$h'(2) = 13$$

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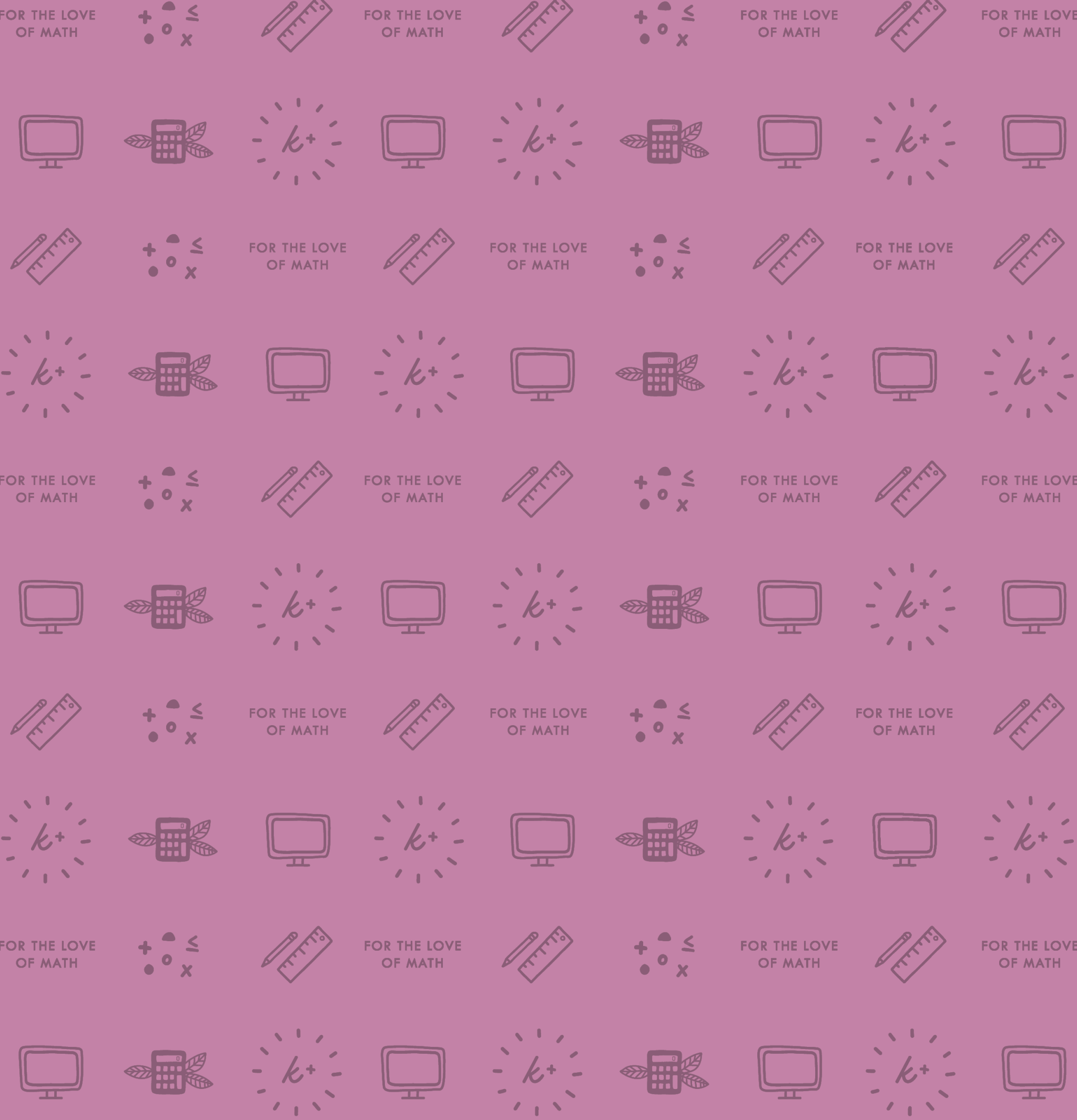

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